example, at the end of the book, Haar's 1910 paper has the number [113], but is cited as [117] in the text. There are also many minor misprints but these cause less inconvenience.

Overall, this is a valuable book containing many deep results.

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G. A. Baker, Jr. and P. Graves-Morris, *Padé Approximants*, 2nd ed., Encyclopedia of Mathematics and Its Applications **59**, Cambridge Univ. Press, Cambridge, 1996, xiv + 746 pp.

This extensive work deals with rational approximants that have become known under the name *Padé Approximants*. They have been studied by Jacobi (1846), Frobenius (1881), and, systematically, by Padé (1892). Moreover many of the generalizations introduced later are incorporated. The main use in the 19th century of what is now called the *Padé table* was its application to analytic number theory (cf., the problem of transcendence of e as treated by Hermite (1873)).

The approximation problem is simple: given two nonnegative integers L and M and a (formal) power series

$$f(z) = \sum_{i=0}^{\infty} c_i z^i,$$

find two polynomials

$$A^{[L/M]}(z) = \sum_{i=0}^{L} a_i z^i, \qquad B^{[L/M]}(z) = \sum_{i=0}^{M} b_i z^i,$$

with $b_0 = 1$, such that

$$A^{[L/M]}(z)/B^{[L/M]}(z) = f(z) + O(z^{L+M+1}). \tag{1}$$

This is clearly a nonlinear problem and the existence and uniqueness of solutions (written as rational functions in their lowest terms) is governed by the nonvanishing of certain Hankel-determinants containing the coefficients of the power series f. The rational functions in (1) are placed in a two-dimensional table referred to as the *Padé table*. The determinants that play an important role (in existence and uniqueness problems, in explicit formulae, in recurrence relations and connections with continued fractions, etc.) are treated in the same way and that table is usually referred to as the *C-table*.

Although the definition using (1) leaves open the possibility that the approximant does not exist for certain pairs (L, M) ("white spots on the table"), this is not a serious shortcoming. When one uses the linearized problem

$$B_{L/M}(z)\, f(z) - A_{L/M}(z) = O(Z^{L+M+1}), \tag{2}$$

the solution always exists (of course we have to drop the condition on the constant coefficient of $B_{L/M}$ now) and, when written as a rational function $A_{L/M}(z)/B_{L/M}(z)$ in its lowest terms, it is unique. Depending on whether $B_{L/M}(0)$ is zero or not, the definition (2) might lead to a *defect* in the order of approximation when compared to definition (1).

Now for the contents of the book—after the introductory chapter (37 pages), a chapter on elementary developments (including some explicit forms for the denominators and a short discussion of the reconstruction of singularities from denominators; 29 pages) the third chapter (56 pages) gives a theoretical treatment of the connection with numerical analysis methods: Aitken's Δ^2 method, convergence acceleration, ε- and η-algorithm, Wynn's identities, recursive calculation of coefficients, Kronecker's and Q. D. algorithm (Quotienten–Differenzen Algoritmus; cf., Rutishauser (1957)), and Cordellier's identity.

As seen from the recurrence relations, there is an intimate connection between continued fractions and sequences of Padé approximants (step lines, diagonals, etc.), treated in Chapter 4 (71 pages, including the Berlekamp–Massey algorithm), followed by Chapter 5 on Stieltjes and Pólya series (those give rise to *normal* Padé tables) and the connection with moment problems and orthogonal polynomials (83 pages). Now there is yet another method lurking in the background for introducing the Padé approximant (cf., Brezinski: *Padé-Type Approximation and General Orthogonal Polynomials*, International Series of Numerical Mathematics **50**, Birkhäuser, 1980). Using the method of linear functionals one arrives at the approximant by introducing orthogonality conditions. The existence of "sufficiently" many orthogonal polynomials—and thus Padé approximants of the right degree/order—follows from the non-vanishing of the determinants already figuring in the C-table. This method is not pursued here.

The sixth chapter deals with convergence theory (pointwise, uniform): along rows, diagonals, and step lines (continued fractions come in handy there). Also generalizations are considered: convergence in measure, in capacity (60 pages).

Then come two chapters that are most important for the reader who wants to have a state of the art overview: Chapter 7 (80 pages) on extensions (multipoint- and Baker–Gammel approximants, Padé–Laurent, Padé–Fourier, Padé–Tchebycheff, Laurent–Padé, multivariate approximants) and Chapter 8 (155 pages) on multiseries (simultaneous approximation, operator approximants, vector-Padé, integral and algebraic approximants). For anyone interested in the subject, this is the heart of the book: an up-to-date account of the existing directions of research on Padé–Hermite approximation; after reading this book one has only to check the reviews since the end of 1996 to find out where the research is today.

The book then concludes with three chapters on applications: the connection with integral equations and quantum mechanics (58 pages), the connection with numerical analysis (46 pages, a.o. Crank–Nicholson and Carathéodory–Fejér), and the connection with quantum field theory (16 pages). Included are also an appendix containing a Fortran FUNCTION call to calculate approximants when a section of the power series is given (less than 200 lines of code), a bibliography (46 pages!), and a short index.

There can only be one conclusion: this book is indispensable to the researcher and the would-be researcher in the field of Padé approximation. Comparing this edition to the first one (in two volumes, 1981) one can only feel great admiration for the authors: they have done a splendid job incorporating all major developments in the theory and application of Padé approximation over the latest 15 years!

It is to be hoped that the reader will treat the examples in this volume as *exercises* (which they were originally in the first edition) that have to be worked: only through hands-on experience can one hope to master the subject.

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S. L. Sobolev and L. Vaskevich, *The Theory of Cubature Formulas*, Mathematics and Its Applications **415**, Kluwer, Dordrecht, 1997, xxi + 416 pp.

The following books dealing with numerical integration in higher dimensions are well known: Davis and Rabinowitz [1], Engels [2], Mysovskikh [3], Sloan and Joe [4], and Stroud [7]. They mainly consider constructive algebraic and number-theoretic topics of cubature. So it is not surprising that this book by Vaskevich and Sobolev on the functional-analytic aspects of cubature mainly presents material which is not covered in the works mentioned above.

Sobolev's first book on cubature [5] appeared in 1974. The translation of the Russian title is *Introduction to Cubature Formulas*. The book mainly touches on functional analysis and gives a new outlook on applications in numerical integration. An English translation [6] was published in 1992